Question1

A ladder, 5 meters long; rests against a vertical wall. If its top slides downwards at the rate of $10 \mathrm{~cm/s}$, then the angle between the ladder and the floor is decreasing at the rate of radians/second when it's lower end is $4 \mathrm{~m}$ away from the wall. MHT CET 2023 (12 May Shift 2)

Options:

A. -0.1

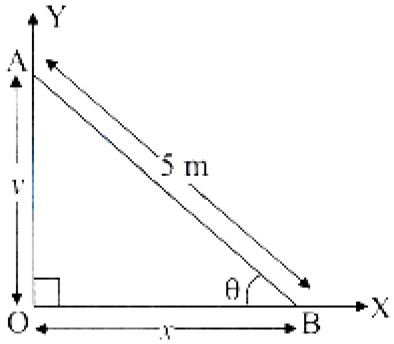
B. -0.025

C. 0.1

D. 0.025

Answer: D

Solution:



According to the figure, $x^2+y^2=25$

Note that $\cos\theta=\frac{OB}{AB}=\frac{x}{5}$

$\therefore \quad x=5\cos\theta$

$\therefore \quad$ (i) $\Rightarrow 25\cos^2\theta+y^2=25$

Differentiating w.r.t. 't', we get

$$-50\cos\theta\sin\theta\frac{d\theta}{dt}+2y\frac{dy}{dt}=0$$
$$25\sin\theta\cos\theta\frac{d\theta}{dt}=y\frac{dy}{dt}$$
$$\therefore \quad 25\sin\theta\cos\theta\frac{d\theta}{dt}=y(-0.1)$$
$$\ldots\left[\because \frac{dy}{dx}=-10\ \mathrm{cm/s}=-0.1\ \mathrm{m/s}\right]$$
$$\therefore \quad 25\sin\theta\cos\theta\frac{d\theta}{dt}=-(0.1)y$$
$$\text{at } x=4, \cos\theta=\frac{4}{5}, \sin\theta=\frac{3}{5} \text{ and } y=3$$
$$\therefore \quad \text{(ii)} \Rightarrow 25\times\frac{3}{5}\times\frac{4}{5}\times\frac{d\theta}{dt}=-0.3$$
$$\Rightarrow \frac{d\theta}{dt}=-0.025$$

i.e., the angle is decreasing at the rate of $0.025\mathrm{rad/s}$

Question2

A ladder 5 meters long rests against a vertical wall. If its top slides downwards at the rate of $10\ \mathrm{cm/s}$, then the angle between the ladder and the floor is decreasing at the rate of rad./s when it's lower end is $4\ \mathrm{m}$ away from the wall. MHT CET 2023 (12 May Shift 1)

Options:

A. -0.1

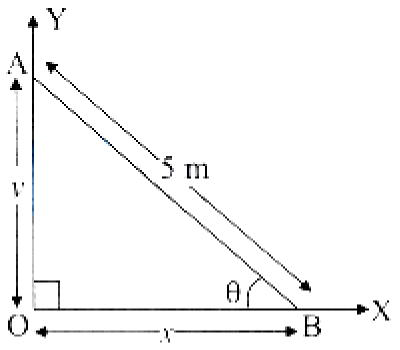
B. -0.025

C. 0.1

D. 0.025

Answer: D

Solution:



According to the figure, $x^2+y^2=25 \ldots(i)$

Note that $\cos\theta=\frac{\mathrm{OB}}{\mathrm{AB}}=\frac{x}{5}$

$\therefore \quad x=5\cos\theta$

$\therefore \quad$ (i) $\Rightarrow 25\cos^2\theta+y^2=25$

Differentiating w.r.t. ' t ', we get

$$
\begin{aligned}
& -50\cos\theta\sin\theta\frac{\mathrm{d}\theta}{\mathrm{dt}}+2y\frac{\mathrm{d}y}{\mathrm{dt}}=0 \\
& 25\sin\theta\cos\theta\frac{\mathrm{d}\theta}{\mathrm{dt}}=y\frac{\mathrm{d}y}{\mathrm{dt}} \\
\therefore \quad & 25\sin\theta\cos\theta\frac{\mathrm{d}\theta}{\mathrm{dt}}=y(-0.1) \\
\therefore \quad & \ldots\left[\because \frac{\mathrm{d}y}{\mathrm{d}x}=-10\ \mathrm{cm/s}=-0.1\ \mathrm{m/s}\right] \\
& 25\sin\theta\cos\theta\frac{\mathrm{d}\theta}{\mathrm{dt}}=-(0.1)y \quad ,.(\mathrm{ii}) \\
& \text{at } x=4, \cos\theta=\frac{4}{5}, \sin\theta=\frac{3}{5} \text{ and } y=3 \\
& (\mathrm{ii}) \Rightarrow 25\times\frac{3}{5}\times\frac{4}{5}\times\frac{\mathrm{d}\theta}{\mathrm{dt}}=-0.3 \\
\Rightarrow & \frac{\mathrm{d}\theta}{\mathrm{dt}}=-0.025
\end{aligned}
$$

i.e., the angle is decreasing at the rate of $0.025 \mathrm{rad/s}$

Question3

A rod AB, 13 feet long moves with its ends A and B on two perpendicular lines OX and OY respectively. When A is 5 feet from O, it is moving away at the rate of $3 \mathrm{feet/sec}$. At this instant, B is moving at the rate MHT CET 2023 (11 May Shift 2)

Options:

A. $\frac{5}{4}\mathrm{ft/sec}$ upwards.

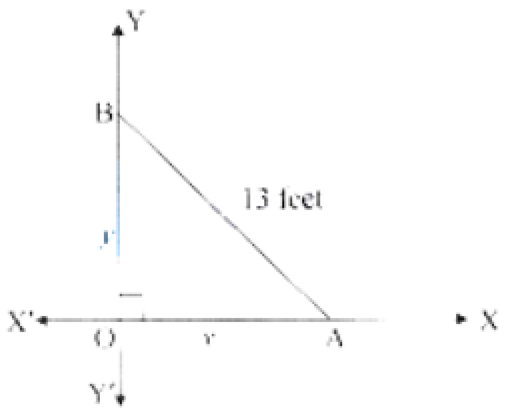
B. $\frac{4}{5}\mathrm{ft/sec}$ upwards.

C. $\frac{5}{4}\mathrm{ft/sec}$ downwards.

D. $\frac{4}{5}\mathrm{ft/sec}$ downwards.

Answer: C

Solution:



Note that $\triangle \mathrm{OAB}$ is a right angled triangle. Let $\mathrm{OA} = x\mathrm{ft}$ and $\mathrm{OB} = y\mathrm{ft}$.

$\therefore \quad y^2 = 169 - x^2$

Now, differentiating above function w.r.t. time 't', we get

$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$

At $x = 5, \frac{dx}{dt} = 3\mathrm{ft/sec}$... [Given]

Also, at $x = 5, y = 12$

$\therefore \quad$ (i) $\Rightarrow 2(12)\frac{dy}{dt} = -2(5)(3)$

$\Rightarrow \frac{dy}{dx} = \frac{-5}{4}$

Negative sign indicates that B is moving downwards.

$\therefore \quad$ B is moving at the rate $\frac{5}{4}\mathrm{ft/sec}$ downwards.
